Synopsis of proposed research work entitled

"Analysis of Integral Transforms and their Applications''

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Ph. D. Synopsis

"Analysis of Integral Transforms and their Applications''

1. Introduction:

The investigation of integral transformations in many nations throughout the world. Namians introduced the fractional Fourier transform in the extended classical transform for the first time in 1980 utilizing the eigen value approach. Zemanian was the first to describe the natural language for generalized functions and singular functions such as Dirac-Delta. Fractional Fourier transformations are used extensively in optics and image processing. Ding and Pei have made significant contributions in this topic, working on the bulk of integral transforms and their crucial applications. The Fourier transform is without a doubt one of the most useful and widely used methods in signal processing and analysis. Fourier transform is used not only in technical subjects but also in other subjects such as music, stock market psychology, economics, and so on. Nevertheless, all scientists select a tiny domain to acquire diverse results for our integral transform and aim to obtain multiple applications in a particular transform.

Without a doubt, the Fourier transform stands out among these integral transforms due to its wide range of applications, particularly in optics, signal processing, image processing, and so on. Moshinky and Quesne presented the linear canonical transform, a considerably more broad integral transform, in 1970. Namias developed the fractional Fourier transform by employing the eigenvalue and eigenfunction technique. Pie and Ding defined fractional cosine transform and fractional sine transform based on the definition of fractional Fourier transform, and they also introduced canonical cosine transform and canonical sine transform as

generalizations of the fractional cosine transform and fractional sine transform. The integral transformations are significant because they give valuable operational approaches for solving initial value issues and initial boundary value problems for linear differential and integral equations. The study of differential and integral equations appearing in applied mathematics, mathematical physics, and engineering sciences was a major effect on the creation of operational calculus of the integral transform. In this examination, we will look at novel transformations and their characteristics, applications, and pictorial representation using MATLAB software.

Differential equations have long played an important part in all aspects of practical mathematics, and their relevance has grown with the introduction of the computer. Consequently, the examination and analysis of differential equations cruising in applications resulted in many complex mathematical difficulties; hence, there are several approaches for solving differential equations. Integral transforms were widely employed to solve differential equations and integral equations, and so there are various words on the theory and use of integral transforms. In mathematics, a transformation is a mechanism that is helpful for converting one function f(t) into another function with a new variable $\mathscr{F}$(s). The integral

Transform of a function f(x) in [a, b], is denoted by $\mathscr{F}$(s)=$\emptyset${f(x)}

$$\mathscr{F}(s)=\emptyset\{f(x)\}=\int_a^b K(x,s)f(x)dx$$

Integral transformations have been utilised successfully in mathematics and engineering for almost two centuries. Integral transform techniques provide a unified mathematical approach to the analysis of electrical, network, energy conversion and control antennas, and electrical system components. Integral transforms were used to investigate electrical transmission through wire or optical fibres, as well as wireless radio propagation. Integral transformations offer science

and engineering with both theory and applications. Integral transform methods are important to engineering education. Despite this, their applications remain prevalent in advanced study and research. As a result of their outstanding work and applicability in nearly every discipline, integral transforms are grabbing the attention of all researchers.

Some useful Definition

- **New General Integral Transform:**

Let (t) be an integrable function defined for $t\geq 0$ $(s)\neq 0$ and $q(s)$ are positive real valued functions then the general integrable transform $\nearrow(s)$ of $f(t)$ is defined by

$$T\{f(t)\} = p(s)\int_0^\infty f(t)\, e^{-q(s)t}dt$$

Provided that the integral exists for some (s).

- **Double new general integral transform:**

Let $f(x,y)$ be an integrable function defined for the variables x and y in the first quadrant, $p1(s)\neq 0, p2(s)\neq 0$ and $q1(s), q2(s)$ are positive real functions; we define the Double new general integral transform $T2\{f(x,y)\}$ by the formula

$$T2\{f(x,y)\}=r(s) = p1(s)p2(s)\int_0^\infty f(x,y)\, e^{-(q1(s)x+q2(s)y)}\, dx\, dy$$

Provided that the integral exists for some $q1(s)$, $q2(s)$

- **Laplace Transform:**

The Laplace Transform is defined as follow

$$L\{f(t)\}= f(s) = \int_0^\infty e^{-st}f(t)dt \ , t > 0$$

- **Tripal Laplace Transform:**

Let f(x,y,t) be a continuous function that can be expressed as convergent infinite series , then triple Laplace transform of f(x,y,t) is defined as

$$L[f(x,y,t)]=F(\sigma,\rho,\delta)=\int_0^\infty\int_0^\infty\int_0^\infty e^{-\sigma x-\rho y-\delta t}\ f(x,y,t)\ dx\ dy\ dt$$

Where x ,y ,t > 0 and σ,ρ,δ are Laplace variables and

- **KUSHARE Transform:**

Kushare transform of function $f(t)$ is denoted by $kf(t)= S(v)$ and it is defined as

$$K[f(t)] =S(v)=v\int_0^\infty f(t)\ e\text{-}tv^a \quad dt,\ t\geq 0$$

Where α is any non-zero real numbers.

2. Review of Literature and development in the subject (Previous work done in related area):

In mathematics, an integral transform maps a function from its original function space into another function space via integration, where some of the properties of the original function might be more easily characterized and manipulated than in the original function space. The transformed function can generally be mapped back to the original function space using the inverse transform. The precursor of the transforms was the Fourier series to express functions in finite intervals. Later the Fourier transform was developed to remove the requirement of finite intervals. Using the Fourier series, just about any practical function of time (the voltage across the terminals of an electronic device for example) can be represented as a sum of sines and cosines, each suitably scaled (multiplied by a constant factor), shifted (advanced or retarded in time) and "squeezed" or "stretched" (increasing or decreasing the frequency). The sines and cosines in the Fourier series are an example of an orthonormal basis. As an example of an, consider the Laplace transform. This is a technique that maps differential or Integro-differential equations. The Laplace transform finds wide application in physics. Integral transforms are closely related to functional series. However, instead of a sum with each term consisting of a coefficient multiplying a basis function, we have an integral in which the summation index is

replaced by an integration variable. The basic functions are multiplied by a function of this integration variable, and integration over this variable yields a representation of the function. This function is called the *integral transform* of the given function and is analogous to the set of expansion coefficients. There are several kinds of integral transforms, including Mellin transforms, Hankel transforms, Laplace Transform, Fourier transform, Sumudu Transform, Sawi Transform, Tarig Transform, Aboodh Transform, Kamal Transforms form, KUSHARE transform, Anuj Transform, double Laplace transform

The application of integral transform in physics, Engineering, Biological sciences, Health sciences and Environmental sciences , particularly in electrical engineering, where the characteristic equations that describe the behavior of an electric circuit. Other integral transforms find special applicability within other scientific and mathematical disciplines. Integral transforms are important to solve real problems. Appropriate choice of integral transforms helps to convert differential equations as well as integral equations into terms of an algebraic equation that can be solved easily. During last two decades many integral transforms in the class of Laplace transform are introduced such as Sumudu, Elzaki, Natural, Aboodh, Pourreza, Mohand, G-transform, Sawi and Kamal transforms

Integral transforms are precious for the simplification that they bring about, most often in dealing with differential equations subject to specific boundary conditions. Appropriate choice of integral transforms helps to convert differential equations and integral equations into terms of an algebraic equation that can be solved easily. The solution achieved is, of course, the transform of the solution of the original differential equation, and it is necessary to invert this transform to complete the operation. Also, use of integral transform is easier technique in solving differential equations and boundary value problems are of three type's

elliptics, parabolic and hyperbolic. Double Laplace transform which can be used to solve fractional partial differential equations that represents many physical and engineering models. The triple Laplace transform for the solution of a class of fractional order partial differential equations.

Convolution theorem for Kushare integral transform and solve convolution type of Volterra integral equation of first kind. Double General Integral Transform for Solving Boundary Value Problems in Partial Differential Equations. We use "KUSHARE Transform" for solving the system of ordinary differential equations of first order and first degree. Many researchers use various integral transforms of error functions to evaluate improper integrals. Integral transforms play important role in solving differential equations and integral equations. application of Aboodh transform and Mahgoub transform in solving boundary value problem of system of ordinary differential equations and result shows that Aboodh transform and Mahgoub transform are closely connected. Apply Double Mahgoub Transform to solve the parabolic boundary value problem. Sawi transform of Bessel's functions with application for evaluating definite integrals.

Integral transforms are important to solve real problems. Appropriate choice of integral transforms helps to convert differential equations as well as integral equations into terms of an algebraic equation that can be solved easily. These integrals transforms have been used for solving different type of integral equations, ordinary differential equations (ODEs), partial differential equations (PDEs) and fractional differential equations (FDEs). A new integral transform is introduced and is used to obtain analytical solution of higher order ODES with constant and variable coefficient and integral equations. Volterra integral Equation :- Volterra Integral equations occurs in variety of branches like Physics, Chemistry, Biology and Engineering. These equations have been solved by using Laplace Transform,

Sumudu Transform, Sawi Transform, Tarig Transform, Aboodh Transform and Kamal Transform. In this paper we use recently developed Anuj Transform for solving linear Volterra integral equations of first kind. Some problems are solved by using Anuj transform, this shows that Anuj transform is useful and effective for solving such equations.

Volterra integral equations has many applications in various fields like demography, study of Viscoelastic material and actuarial science through the renewal equations, are some of the fields in October 2021 S. S. Khakale and D. P. Patil introduced Soham transform. Anuj transform is introduced by Anuj Kumar, Shikha Bansal and Sudhanshu Aggarwal in January 2022. In September 2021 Kushare and Patil introduced Kushare transform, for simplifying the process of obtaining solution of ordinary and partial differential equations in the time domain.

3. Objectives of Research /Proposed Hypothesis:

The main objectives of this study are:

1. To study in details some integral transform.
2. To generalize new integral transform.
3. To apply new integral transform for solving the problems and study their applications in different relevant field.
4. To study the existing methods.
5. To study the applications with relevant/ engineering field.

4. Methodology to be adopted:

1. Collection of relevant research paper, review articles, books, journals etc. through internet and by visiting various libraries.

2. To participate in the relevant fields like conferences, workshops, seminars.
3. To obtain an alternative solution of the problems by applying new technique and compare the results with the results already obtained.
4. To use computational software like Maple, Mathematica, Mathcad, for numerical calculations.
5. To find out applications in the relevant field.
6. To publish the work in the reputed journals.

5. Importance of Study/Society Application:

Interdisciplinary Relevance:

The issue has great importance because it is an interdisciplinary work related with the branch of science and technology like Physics, Mechanical Engineering, and Civil Engineering etc. The isotropic and anisotropic thermoelastic problems to be considered have important industrial applications.

International & National Status:

Lot of work has been carried out by many National and International researchers on thermoelasticity. The works of few amongst them are quoted in the direct and indirect problems. (**Refer Origin of the Research Problem**).

6. Proposed Work Plan/Formulation & Structure of Study:

Year-Wise plan of work and targets to be achieved:

First year

i) To collect the references in relevant field.

ii) To study the references in relevant field.

iii) To attend the seminars/Conferences/Workshops on relevant topic.

Second year

i) To attend the seminars/Conferences/Workshops on relevant topic.

ii) To formulate new integral transform.

iii) To study existing integral transform which will be applied on these problems.

iv) To construct the mathematical models for various integral transform

Third year

i) To study applications of obtained results

ii) To publish the work in the reputed journals.

iii) To submit the work to the authority.

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